Tutorial 9: Binary Response Models

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Binary Outcomes

- Model binary decision making of economic agents

  ↳ Firm enter or exit market (IO)
  Work in private or pubic (Labour)
  Register for Econometrics (Education)

- Example: decision to attend graduate school after undergrad?

  $$\hookrightarrow Y_i = \begin{cases} 1, & \text{student } i \text{ pursues grad. school} \\ 0, & \text{otherwise} \end{cases}$$

- Select decision to maximize utility

  Let $U_i(Y_i)$ be utility from $Y_i \in \{0,1\}$

  ↳ $Y_i = 1$ if $U_i(1) > U_i(0)$

Model of Binary Decision Making

- $U_i(Y_i; X_i)$ is utility from selecting $Y_i \in \{0, 1\}$

  $\hookrightarrow Y_i \in \{HS, College\}$, X_i = Parents Educ. $\Rightarrow U_i(Y_i, X_i)$

* Cannot observe utility but infer difference based on observed choices

- Parameterize $U_i(Y_i; X_i) = Y_i(\beta_0 + \beta_1 X_i - \epsilon_i)$

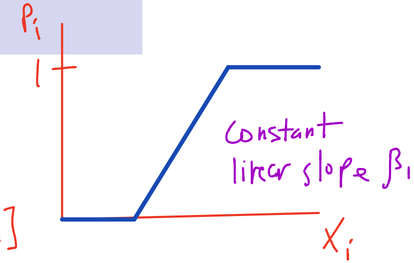
$$= \begin{cases} \beta_0 + \beta_1 X_i - \varepsilon_i, & Y_i = 1 \\ 0, & Y_i = 0 \end{cases}$$

- Select $\cancel{d_i}$ $Y_i = 1$ if $U_i(1; X_i) > U_i(0; X_i)$

$$Y_i = 1 \Rightarrow U_i(1; X_i) - U_i(0; X_i) > 0 \Rightarrow \beta_0 + \beta_1 X_i - \varepsilon_i > 0$$

$$\Pr(Y_i = 1 | X_i) = \Pr(\beta_0 + \beta_1 X_i - \varepsilon_i > 0) = \Pr(\varepsilon_i < \beta_0 + \beta_1 X_i) = \underset{\varepsilon \sim F_\varepsilon}{\underset{\downarrow}{F_\varepsilon(\beta_0 + \beta_1 X_i)}}$$

Linear Probability Model



- Assume $\epsilon \sim U[0,1]$

↳ $Pr(Y_i=1|X_i) = F_\epsilon(\beta_0+\beta_1 X_i) = \begin{cases} 1, & \beta_0+\beta_1X_i > 0 \\ \beta_0+\beta_1X_i, & \beta_0+\beta_1X_i \in [0,1] \\ 0, & \beta_0+\beta_1X_i < 0 \end{cases}$

- LPM: $Y_i = \beta_0 + \beta_1 X_i + u_i$

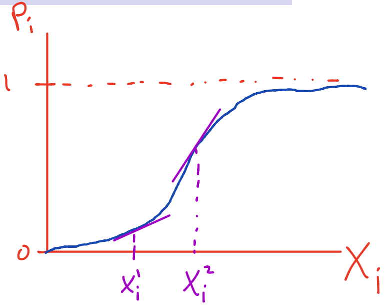
↳ $E(Y_i|X_i) = \beta_0+\beta_1X_i \Rightarrow$ Since $Y_i \in \{0,1\} \Rightarrow E(Y_i|X_i) = Pr(Y_i=1|X_i)$

- Marginal effect: $\frac{dPr(Y_i=1|X_i)}{dX_i} = \beta_1$

↳ Assume X_i cont.

↳ Unit increase in X, that is associated with β_1 increase in Prob. $Y_i=1$, on average.

Probit Model



- Assume $\epsilon \sim N(0, \sigma_e^2)$

↳ $Pr(Y_i = 1|X_i) = F_\varepsilon(\beta_0 + \beta_1 X_i) = \Phi(\beta_0 + \beta_1 X_i)$

- $Pr(Y_i = 1|X_i) = \Phi(\beta_0 + \beta_1 X_i)$

↳ Challenge: Cannot direct interpret $\hat{\beta}_1$

↳ β_1 non-linearly related to Y_i

- Marginal effect: $\frac{dPr(Y_i=1|X_i)}{dX_i} = \phi(\beta_0 + \beta_1 X_i)\beta_1$ → marginal effect varies with X

↳ chain rule ⇒ $\frac{dPr(Y_i=1|X_i)}{dX_i} = \underbrace{f_\varepsilon}_{\text{pdf}}(\beta_0 + \beta_1 X_i) \times \beta_1$ → Evalue marginal effect at $\overline{X}$

Logit Model

Similar to the probit model
↳ Also has "S-shaped" curve for $Pr(Y_i=1|X_i)$

- Assume ϵ has a logit distribution

*In many cases LPM, Logit, Probit give similar results.

- $Pr(Y_i = 1) = \frac{exp(\beta_0+\beta_1 X_i)}{1+exp(\beta_0+\beta_1 X_i)}$

- Marginal effect: $\frac{dPr(Y_i=1|X_i)}{dX_i} = \beta_1 \frac{exp(\beta_0+\beta_1 X_i)}{[1+exp(\beta_0+\beta_1 X_i)]^2}$