Tutorial 2: Simultaneous Equations

Hammad Shaikh

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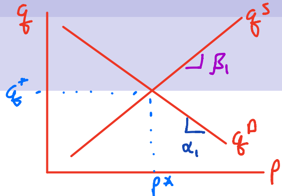
Simultaneous Equations Overview



Data: (p^*, q^*)

Eqns associated with economic theory

- Representing economic relationships using multiple equations

Structural Eqns
$$\begin{cases} p = \beta_0 + \beta_1 q_s + \beta_2 z_1 + u_1 & \text{(supply)} \\ p = \alpha_0 + \alpha_1 q_d + \alpha_2 z_2 + u_2 & \text{(Demand)} \\ q_s = q_d = q & \text{(Eqbm)} \end{cases}$$

- Endogeneity problem if estimate equations on their own

Shown that $Cov(q, u_1) \neq 0$, $Cov(q, u_2) \neq 0$ ("simultaneity bias")

↳ $p \rightarrow q$ & $q \rightarrow p$ $\Rightarrow$ OLS being biased & inconsistent

- Identification of a equation requires at least as many excluded IVs as endogenous variables

↳ Need IV for $\underset{z_2}{q_s}$ and $\underset{z_1}{q_d}$

Recall: IV
$$\begin{cases} \text{Relevant: } Cov(q, z) \neq 0 \\ \text{Exog: } Cov(u, z) = 0 \\ \text{Exclusion: } z \text{ is not included in the eq}^n \text{ with } q \end{cases}$$

Simultaneous Equations Practice

Chapter 16 Problem 1

Notation $\begin{cases} \text{Exog: } z_1, z_2 \\ \text{Endog: } y_2 \\ \text{Error: } u_1, u_2 \end{cases}$

If $\alpha_1 \neq 0, \alpha_2 \neq 0$, and $\alpha_1 \neq \alpha_2$, find reduced form for y_1 and y_2

Structural Eqns
$$
\begin{cases} y_1 = \alpha_1 y_2 + \beta_1 z_1 + u_1 & (1) \\ y_1 = \alpha_2 y_2 + \beta_2 z_2 + u_2 & (2) \end{cases}
$$

Is y_2 correlated with u_1 and u_2?

$(1) \Rightarrow y_2 = \frac{1}{\alpha_1}(y_1 - \beta_1 z_1 - u_1) \xrightarrow{(2)} y_1 = \frac{\alpha_2}{\alpha_1}(y_1 - \beta_1 z_1 - u_1) + \beta_2 z_2 + u_2$

$\Rightarrow$ solve for

$$
\left.\begin{aligned} y_1 &= \frac{\alpha_1 \alpha_2}{\alpha_1 - \alpha_2}\left(\frac{\beta_2}{\alpha_2} z_2 + \beta_1 z_1 + \frac{u_2}{\alpha_2} + u_1\right) \\ y_2 &= \frac{1}{\alpha_1 - \alpha_2}(\beta_2 z_2 - \beta_1 z_1 + u_2 - u_1) \end{aligned}\right\} \text{Reduced form}
$$

$\hookrightarrow Cov(y_2, u_1) \neq 0, \; Cov(y_2, u_2) \neq 0$

$\hookrightarrow$ OLS for (1) & (2) is biased

Simultaneous Equations Practice

Chapter 16 Problem 3

In Problem 3 of Chapter 3, we estimated an equation to test for a tradeoff between minutes per week spent sleeping (sleep) and minutes per week spent working (tot wrk) for a random sample of individuals. We also included education and age in the equation. Because sleep and tot wrk are jointly chosen by each individual, is the estimated tradeoff between sleeping and working subject to a "simultaneity bias" criticism? Explain.

$$totwrk_i = \beta_0 + \underbrace{\beta_1}_{\text{param. of interest}} sleep_i + \beta_2 educ_i + \beta_3 age_i + \varepsilon_i$$

Sleep $\rightarrow$ awareness $\rightarrow$ productivity $\rightarrow$ Hours Worked

$\hookrightarrow$ Sleep $\rightarrow$ Hours Worked (Reasonable)

Hours Worked $\xrightarrow{?}$ Sleep only holds in extreme cases

eg. Bob works 20 hours, but for most data this direction likely is not strong

$\Downarrow$

No "simutaneity bias"

Simultaneous Equations Practice

Chapter 16 Problem 4

Assume education and price are exogenous. Which equations are identified?

$$log(earnings) = \beta_0 + \beta_1 alcohol + \beta_2 educ + u_1 \quad (1)$$

$$alcohol = \gamma_0 + \gamma_1 log(earnings) + \gamma_2 educ + \gamma_3 log(price) + u_2 \quad (2)$$

Identification
- Need IV for alcohol to identify (1)
- Need IV for earnings to identify (2)

↳ (1) is identified
- Price IV for alcohol
  ↳ i) Relevance: $Cov(price, alcohol) \neq 0 \iff \gamma_3 \neq 0$
  ii) Exog: $Cov(price, u_1) = 0$
  iii) Exclusion: price not in (1)

(2) not identified since no valid excludible IV for earnings

Simultaneous Equations Practice

$\begin{cases} \text{Exog}: z_1, z_2, z_3 \\ \text{Endog}: y_1, y_2, y_3 \end{cases}$

Which equations are identified?

$$y_1 = \alpha_0 + \alpha_1 y_2 + \alpha_2 y_3 + \alpha_3 z_1 + \alpha_4 z_2 + u_1 \quad (1)$$

$$y_2 = \beta_0 + \beta_1 y_1 + \beta_2 z_2 + u_2 \quad (2)$$

$$y_3 = \gamma_0 + \gamma_1 z_1 + \gamma_2 z_3 + u_3 \quad (3)$$

① Two endog. y_2, y_3, need at least 2 IVs, only z_3 excluded exog. variable $\Rightarrow$ not identified $\Rightarrow$ no consistent estimator for α_1, α_2

② Either z_1, z_3 as IV for y_1 $\Rightarrow$ identified

③ 0 endog. var. $\Rightarrow$ identified (OLS)

Simultaneous Equations Practice

$\begin{cases} \text{Exog: } z_1, z_2, z_3, z_4, z_6 \\ \text{Endog: } y_1, y_2, y_3, y_4 \end{cases}$

Which equations are identified?

$$y_1 = \alpha_{11} y_2 + \alpha_{12} y_3 + \alpha_{13} z_1 + u_1 \tag{1}$$

$$y_2 = \alpha_{21} y_1 + \alpha_{22} z_2 + \alpha_{23} z_3 + u_2 \tag{2}$$

$$y_3 = \alpha_{31} y_2 + \alpha_{32} y_4 + \alpha_{33} z_1 + \alpha_{34} z_2 + \alpha_{35} z_4 + \alpha_{36} z_6 + u_3 \tag{3}$$

$$y_4 = \alpha_{41} y_3 + \alpha_{42} z_1 + u_4 \tag{4}$$

(1) has two endog. y_2, y_3, z_2, z_3 IV y_2, z_2, z_4, z_6 IV for y_3

↳ more than two excluded IVs ⇒ identified

(2) one endog. y_1, z_1 IV for y_1 ⇒ identified

(3) two endog. y_2, y_4, includes four exog. z_1, z_2, z_4, z_6

↳ Only z_3 excluded ⇒ not identified

(4) z_2, z_4, z_6 IV for y_3 ⇒ identified