Tutorial 4: Time Series Seasonality and Stationarity

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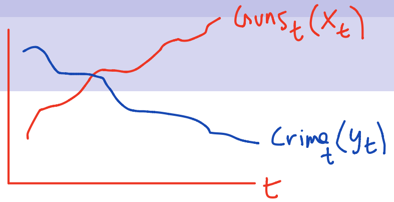
Seasonal Adjustments



⇒ Regress $crime_t$ on $Guns_t$
⇒ $\hat{\beta}_1 < 0$
↳ Could be case that gun ownership does not cause crime, but both have a trend.

- Challenge: series tend to trend with seasons and time

  ↳ Need to isolate seasonality & trends to avoid spurious regression results

- Include time trends in regression

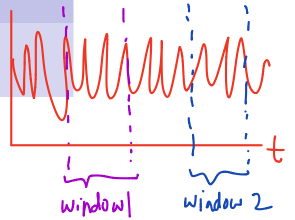
  ↳ $y_t = \beta_0 + \beta_1 X_t + \underbrace{\beta_2 t}_{\text{linear trend}} + \varepsilon_t$

- Include time indicator variables in regression

  ↳ $y_t = \beta_0 + \beta_1 X_t + \underbrace{\delta_{t^*}}_{\text{seasonal fixed effects}} + \varepsilon_t$, t = monthly level across years, t^* = Quarter of year

If Stationarity & WD hold ⇒ relax OLS assump.

Stationary and Weakly Dependant Time Series



stationarity implies both windows of equal lenght realized from same dbn.

Strong Stationarity

- Stationary: joint distribution of stochastic process is stable over time $F_X(X_{t_1}, X_{t_2}, \dots, X_{t_n}) = F_X(X_{t_{1+h}}, X_{t_{2+h}}, \dots, X_{t_{n+h}})$

  ⇒ same statiscal charac. across equal size windows

  ↳ Constant mean, variance, skewness, etc.

Weak Stationarity

- Covariance stationary: finite mean, finite variance, and $Cov(x_t, x_{t+h})$ only depends on h

  ↳ $E(X_t) = \mu$, $V(X_t) = \sigma^2$ and $Cov(X_t, X_{t+h}) = g(h)$

- Weakly dependant: $Cov(x_t, x_{t+h}) \to 0$ has $h \to \infty$

  ↳ Replacing "independent" assump. from cross-sectional data ⇒ Enables for the usage of LLN & CLT for inference

Autoregressive Model and Examples

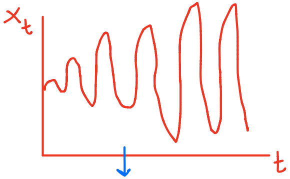
If $\rho = 1 \Rightarrow V(y_t) = \infty$ & $Corr(y_t, y_{t+h}) = 1 \Rightarrow$ not (S) & not (WD)

↳ Similar argument for $\rho = -1$

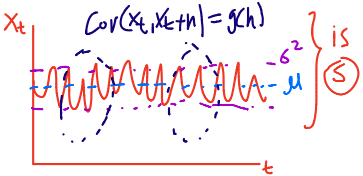
- AR(1) model: $y_t = \rho y_{t-1} + u_t$, $AR(2): y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + u_t$, $u_t \sim N(0, \sigma_u^2)$

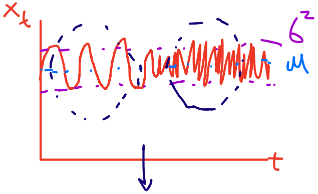
↳ show $E(y_t) = 0$, $V(y_t) = \dfrac{\sigma_u^2}{1-\rho^2}$, $Corr(y_t, y_{t+h}) = \rho^h \Rightarrow$ stationarity & WD requires $\rho \in (-1, 1)$

- Which one of the following time series is stationary?

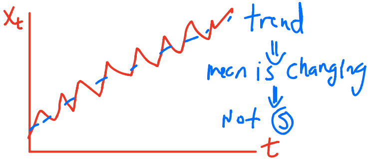


mean constant but var ↑ ⇒ not (S)





Covariance structure changes with time ⇒ not (S)



Seasonal adjustment practice

Chapter 10 Problem 5

Suppose you have quarterly data on new housing starts, interest rates, and real per capita income. Specify a model for housing starts that accounts for possible trends and seasonality in the variables.

$$HStart_t = \beta_0 + \beta_1 int_t + \beta_2 rpci_t + \underbrace{\beta_3 Q2_t + \beta_4 Q3_t + \beta_5 Q4_t}_{\text{seasonality}} + \underbrace{\beta_6 t}_{\text{trend}} + u_t$$

Seasonal adjustment practice

Chapter 11 Problem 2

$e_t \sim iid(0,1) \Rightarrow e_t$ stationary ✓

Let $\{e_t : t = -1, 0, 1, \ldots\}$ be a sequence of independent, identically distributed random variables with mean zero and variance one. Define a stochastic process by

$$x_t = e_t - (1/2)e_{t-1} + (1/2)e_{t-2}, t = 1, 2, \ldots$$

1. Find $E(x_t)$ and $Var(x_t)$. Do either of these depend on t?

   $E(x_t) = E(e_t) - \frac{1}{2}E(e_{t-1}) + \frac{1}{2}E(e_{t-2}) = 0$

   $V(x_t) = V(e_t) + \frac{1}{4}V(e_{t-1}) + \frac{1}{4}V(e_{t-2}) = 1 + \frac{1}{4} + \frac{1}{4} = \frac{3}{2}$

   } not depend on t

2. What is $Corr(x_t, x_{t+h})$ for $h > 2$?

   $Corr(x_t, x_{t+h}) = \frac{Cov(x_t, x_{t+h})}{\sqrt{V(x_t)V(x_{t+h})}}$, $Cov(\underbrace{e_t - \frac{1}{2}e_{t-1} + \frac{1}{2}e_{t-2}}_{x_t}, \underbrace{e_{t+h} - \frac{1}{2}e_{t+h-1} + \frac{1}{2}e_{t+h-2}}_{x_{t+h}}) \underset{h>2}{=} 0$

3. Is x_t an asymptotically uncorrelated process?

   As $h \to \infty \Rightarrow Corr(x_t, x_{t+h}) \to 0 \Rightarrow$ yes