Tutorial 5: Time Series Transformations and Statistical Inference

Hammad Shaikh

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Newey-West Standard Errors

$$\begin{cases} V(u_t|X_t) = \sigma_u^2 & \text{(Homoscedasticity)} \\ Cov(u_t, u_s | X_t, X_s) = 0 & \text{(No serial correlation)} \end{cases}$$

Relevant for Hypo. testing

$H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

$$t = \frac{\hat{\beta}_1 - 0}{SE(\hat{\beta}_1)}$$

- Standard errors from OLS only valid if homoscedasticity and no serial correlation assumptions hold

$\hookrightarrow$ SE from OLS $\Rightarrow$ $Var(\hat{\beta}_{OLS}) = \dfrac{\sigma_u^2}{\sum_t (X_t - \bar{X})^2}$

$y_t = \beta_0 + \beta_1 X_t + u_t$

- Newey-West standard errors are robust to hetroscedasticity and autocorrelation up to lag g

$\hookrightarrow$ $V^{NW}(\hat{\beta}_{OLS}) = Var(\hat{\beta}_{OLS}) \cdot \underbrace{\hat{V}(g)}_{\geq 0,\ \text{Newey-West correction}}$

$\hookrightarrow$ $g = 4\left(\frac{T}{100}\right)^{2/9}$

Random Walk and Moving Average

$u_t \sim iid(0, \sigma_u^2)$

not (S) & not (WD)

- Random walk: $y_t = y_{t-1} + u_t$ (AR(1) with $\rho = 1$)

  $\hookrightarrow y_t = \underbrace{y_0}_{0} + u_1 + u_2 + \dots + u_t \Rightarrow \mathrm{Corr}(y_t, y_{t+h}) = \sqrt{\frac{t}{t+h}} \xrightarrow[h\to\infty]{} 0$ slowly if t large $\Rightarrow$ not (WD)

  $\hookrightarrow \begin{cases} E(y_t) = 0 \\ V(y_t) = V(u_1) + \dots + V(u_t) = t\sigma_u^2 \end{cases} \Rightarrow$ not (S)

- Random walk with drift: $y_t = \beta_0 + y_{t-1} + u_t$

  $\hookrightarrow y_t = \beta_0 + [\beta_0 + y_{t-2} + u_{t-1}] + u_t = t \cdot \beta_0 + \sum_{s=1}^{t} u_s$

  $\hookrightarrow \begin{cases} E(y_t) = t \cdot \beta_0 \\ V(y_t) = t \cdot \sigma_u^2 \end{cases} \Rightarrow$ not (S)

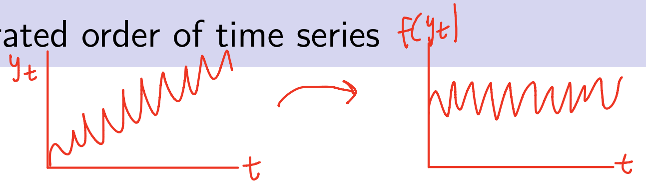
- Moving average MA(1): $y_t = u_t + \alpha_1 u_{t-1}$, MA(2): $y_t = u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2}$

  $\hookrightarrow \begin{cases} E(y_t) = E(u_t) + \alpha_1 E(u_{t-1}) = 0 \\ V(y_t) = V(u_t) + \alpha_1^2 V(u_{t-1}) = \sigma_u^2 + \alpha_1^2\sigma_u^2 = \sigma_u^2(1+\alpha_1^2) \end{cases} \Rightarrow$ (S)

  $\mathrm{Corr}(y_t, y_{t+h}) = 0$ as $h \to \infty \Rightarrow$ (WD)

Integrated order of time series

In practice, most time series are $I(0)$, $I(1)$, or $I(2)$.



- Can transform non-stationary time series to be stationary

  ↳ y_t nonstatationary ⇒ possible $\underbrace{\Delta y_t}_{y_t - y_{t-1}}$ or $\Delta y_t - \Delta y_{t-1}$ are stationary

- Weakly dependant time series are $\underbrace{I(0)}_{\text{No diff. needed for WD}}$

  $MA(0): y_t = u_t + \alpha u_{t-1} \Rightarrow WD \Rightarrow I(0)$

- Time series weakly dependant after first differencing are $\underbrace{I(1)}_{\text{first diff. WD}}$

  $RW: y_t = y_{t-1} + u_t$

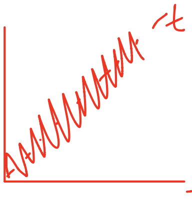
  ↳ $\Delta y_t = y_t - y_{t-1} = u_t \Rightarrow \begin{cases} E(\Delta y_t) = 0 \\ V(\Delta y_t) = \sigma_u^2 \\ Corr(\Delta y_t, \Delta y_{t+h}) = 0 \text{ as } h \to \infty \end{cases} \Rightarrow \underset{S}{WD} \Rightarrow I(1)$

$u_t \sim iid(0,\sigma_u^2)$

Order of Integration Practice

* How many times to take diff. to make WD & S?

What is the order of integration for the following time series?



1. $y_t = u_t$ (white noise)

   $\hookrightarrow \begin{cases} E(y_t)=0 \\ V(y_t)=\sigma_u^2 \\ Corr(y_t, y_{t+h})=0 \text{ as } h\to\infty \end{cases} \Rightarrow$ S & WD $\Rightarrow$ $I(0)$

2. $y_t = \alpha t + u_t$ (trend stationary)

   $\hookrightarrow \begin{cases} E(y_t)=\alpha t \\ V(y_t)=\sigma_u^2 \end{cases} \Rightarrow$ not S $\Rightarrow \Delta y_t = y_t - y_{t-1} = \alpha t + u_t - [\alpha(t-1)+u_{t-1}] = \alpha + u_t - u_{t-1} \Rightarrow$ $E(\Delta y_t)=\alpha$, $V(\Delta y_t)=2\sigma_u^2$, $Corr(\Delta y_t, \Delta y_{t+h})=0$ as $h\to\infty$ $\Rightarrow$ Δy_t is S & WD $\Rightarrow$ $I(1)$

3. $y_t = 2y_{t-1} - y_{t-2} + u_t$ (AR(2))

- y_t like RW $\Rightarrow$ not S
- $\Delta y_t = y_t - y_{t-1} = y_{t-1} - y_{t-2} + u_t \Rightarrow$ not S
- $\Delta y_t - \Delta y_{t-1} = y_t - 2y_{t-1} + y_{t-2} = u_t$

  $\hookrightarrow$ $\Delta y_t - \Delta y_{t-1}$ is S & WD

  $\hookrightarrow$ y_t is $I(2)$

RW-Drift

$$y_t = \beta_0 + y_{t-1} + u_t \Rightarrow \Delta y_t = y_t - y_{t-1} = \beta_0 + u_t$$

$$\Rightarrow E(\Delta y_t) = \beta_0$$

$$V(\Delta y_t) = \sigma_u^2$$

$$Corr(\Delta y_t, \Delta y_{t+h}) = 0 \text{ as } h \to \infty$$

RW-Drift $\Rightarrow I(1)$

$$y_t = \beta_0 t + \sum u_t \Rightarrow \Delta y_t = y_t - y_{t-1}$$

$$= \beta_0 t + \sum_{s=1}^{t} u_s - \left[\beta_0 (t-1) + \sum_{s=1}^{t-1} u_s\right]$$

$$= \beta_0 + \underbrace{\sum_{s=1}^{t} u_s - \sum_{s=1}^{t-1} u_s}_{u_t} = \beta_0 + u_t$$