Tutorial 6: Time Series Unit Root and Cointegeration

Hammad Shaikh

February 25, 2021

Unit Root Process



• Unit root processes can result in spurious relationships

Dickey - Fuller Test for Unit Roots

Assume true • Test for a unit root process: $H_0: \rho = 1, H_1: \rho < 1$ by yt = fyt-1 + Ut, test whether p is 1 Challenge: Under Ho => yt RW => OLS not appropriate for p • Dickey-Fuller test involves: $\Delta y_t = \theta_0 + \theta_1 y_{t-1} + u_t$ Transformation: $y_{t-1} = (p-1)y_{t-1} + u_t \Rightarrow \text{ if } p=1 \Rightarrow \Delta y_t \text{ is stationary } OLS appropriate$ • Reject H_0 (unit root) if $t_{stat}^{DF} < t_{cv,\alpha}^{DF}$ $H_0: \Theta_1 = O$ (unit root) => Estimate $\hat{\Theta}_1 => t_{stat}^{OF}$ $H_1: \Theta_1 < O$ (stationary) ٥F FTR H. RJ. HO 3/5

Cointegeration

• Consider two I(1) time series y_t and x_t EX. is RW $M_{x_{t}}^{Y_{t}} - I(t)$

yt and xt are co-integrated if yt - βxt is I(0)
 yt = βxt + yt => yt = yt - βxt
 b G-integration => there exist β so Ut is I(0)

 $MMMM u_{t} - I(\omega)$

• Co-integeration implies OLS of y_t and x_t is appropriate Exception to when OLS can be meaning full

even when Xt & Yt are I(1)

Engle - Granger Test for Cointegeration

• Let
$$u_t = y_t - \beta x_t$$
, where x_t and y_t are $I(1)$
 $C_{ointegeration} \rightarrow U_t$ is $I(0)$
 $y_t \rightarrow x_t$
• Want to test whether u_t is $I(0)$ with β unknown
 $U_t \in Stimate \ \hat{\beta}$ and $\omega_{nsider} \ \hat{U}_t = y_t - \hat{\beta} x_t$
 $U_t = \int \delta x_t + \delta x_t$
• Estimate $\Delta \hat{u}_t = \gamma_0 + \gamma_1 \hat{u}_{t_1}$ and test $H_0: \gamma_0 = 0$ (unit root),
 $H_1: \gamma_1 < 0$ (not unit root)
Transform
 $U_t = \int \hat{U}_{t-1} + \xi_t \rightarrow H_0: Y_1 < 0$ (stationary)
• Let $u_t = y_t - \hat{\beta} x_t$
 $U_t = \int \hat{U}_{t-1} + \xi_t$